Lecture 9 Network Congestions Part1

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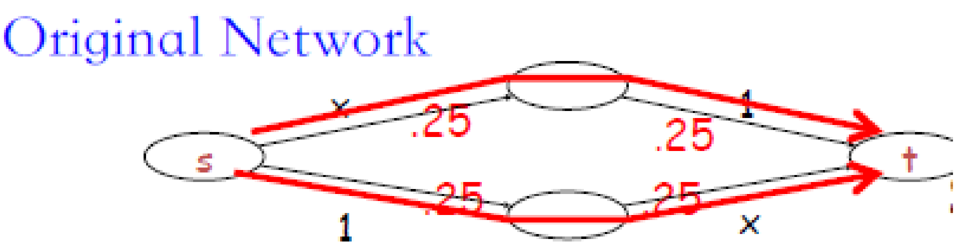
Braess' Paradox

Braess' Paradox is a proposed explanation for the situation where an alteration to a road network to improve traffic flow actually has the reverse effect and impedes traffic through it.
As shown below, time function for someone travelling on a route is affected by the volume of traffic (x) on each link.

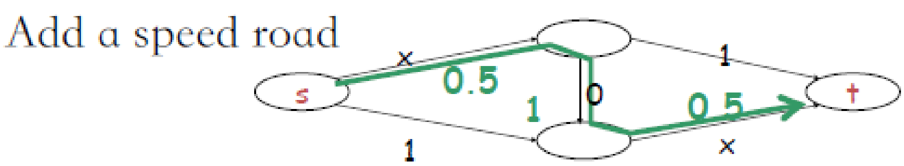
when the traffic=0.5

1.Original Network



Social Consumption at Nash equilibrium(half and half)=0.25*0.25*2+0.25*1*2=0.625

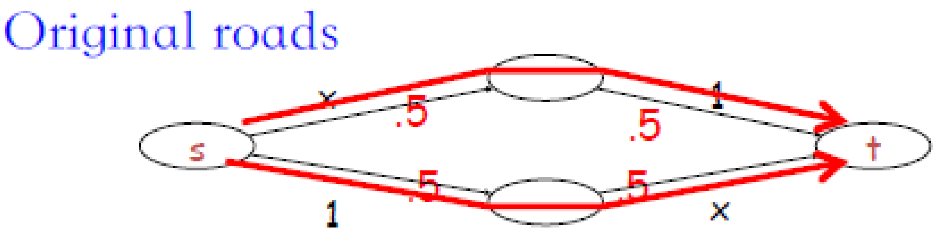
2.Add a fast road



Social Consumption at Nash equilibrium(green line)=0.5*0.5*2=0.5<0.625.So all runs fast and minimize total cost.

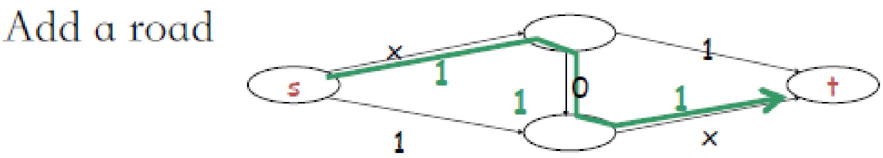
when the traffic=1

1.Original Network



Social Consumption at Nash equilibrium(half and half)=0.5*0.5*2+0.5*1*2=1.5

2.Add a fast road

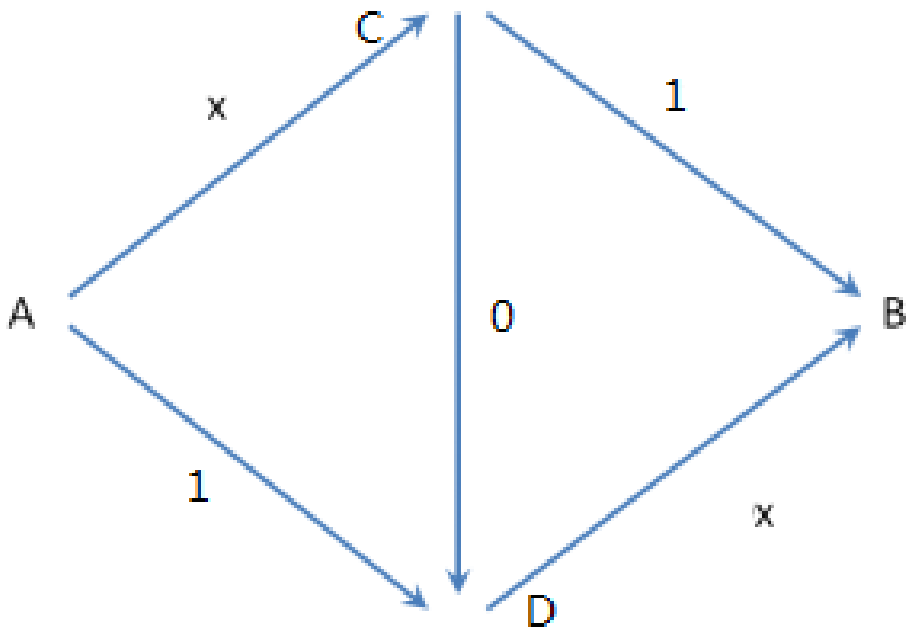


Social Consumption at Nash equilibrium(green line)=1*1*2=2>1.5. Traffic jam is worsen!(called Braess' Paradox)

Nash equilibrium in general cases

1.Original Network(No fast road)
Nash equilibrium: always half and half no matter what the volume of traffic is.

2.Add a fast road as shown below



case 1: when the volume of traffic $T \in (0,1)$
Nash equilibrium: the whole traffic take the path A-C-D-B

case 2: when the volume of traffic $T \in (1,2)$
Nash equilibrium: Suppose the traffic on AC is m and the traffic on CD is n, then the traffic on AD, CB, DB respectively are $T-m, m-n, T-m+n$.
When it is an equilibrium, the time to get from A to B on all the possible routes (ACB , ADB and ACDB) are equal, then we have:

$$\begin{aligned} m+1 &= 1+T-m+n \\ &= m+T-m+n \end{aligned}$$

we can get $m=1$ and $n=2-T$.

case 3: when the volume of traffic $T \in (2,+\infty)$
Nash equilibrium: half and half on ACD and ADB.

Resolution of Traffic Dilemma
-(Pigouvian Tax)
f*l(f):traffic times the first derivative of delay function
-Result:
Nash equilibrium=Social optimum
- Effect
1.total delay minimized
2.no investment but improved revenue
3.individual optimum aligned with social optimum

Issues Involved with Equilibrium
-Social Optimal = outcome maximize the social welfare
Social Welfare: total utility of all players (the sum of individual utilities).

-Nash = outcome of individual rational (selfish) behavior
Nash may be inefficient in terms of social welfare.