Bandwidth Sharing in P2P Networks

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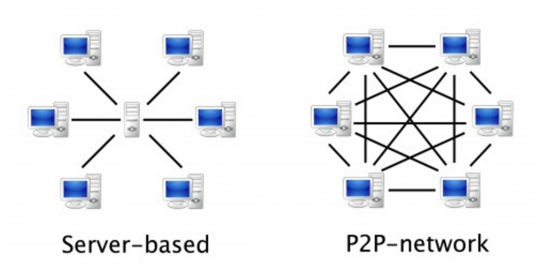
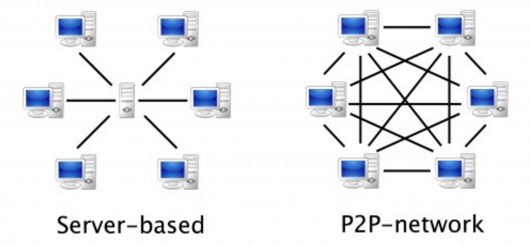
A P2P Model

Cooperation at the Internet



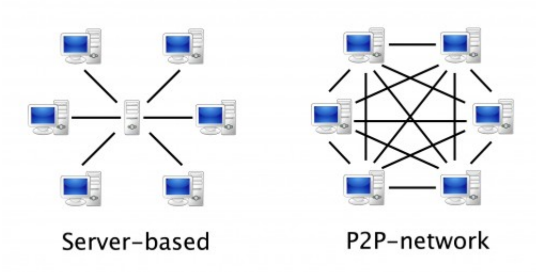
1. Peer-to-Peer
2. Crowd Sourcing

P2P Network Sharing



1. File Sharing
2. Knowledge Sharing
3. Information Sharing

Incentive Design in P2P Systems



eg. BitTorrent.

"One of the main reasons of the recent success of P2P file sharing systems is its built-in tit-for-tat mechanism." (Wu and Zhang STOC2007)

Keywords: "fairness" "efficiency"

P2P Bandwidth Sharing Problem

Formally,
consider a model of an undirected graph $G = (V, E; w)$.

1. Vertex $v \in V$ represents a player (peer);
2. $w_v : V \rightarrow R_+$: the bandwidth of player v which it must upload to its neighbors;
3. x_{vu}: the fraction of bandwidth player v uploads to u for $(v, u) \in E$.
4. $x_{vu} w_v$: the amount of bandwidth player v uploads to u.
5. $X = (x_{vu})_{(v,u) \in E}$: the bandwidth allocation.

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Market Equilibrium Solution

Proportion response protocol

Definition:

For player v, the allocation $(x_{vu} : u \in \Gamma(v))$ of w_v is proportional to what it receives, $(w_k \cdot x_{kv} : k \in \Gamma(v))$,from its neighbor set $\Gamma(v)$.

$$\forall u \in \Gamma(v): \; x_{vu} = \frac{x_{uv} \cdot w_u}{\sum_{k \in \Gamma(v)} x_{kv} \cdot w_k}.$$

An economy model of P2P

Naturally, P2P system can be modeled as a pure exchange economy.

- player v sells its own bandwidth to its neighbors;
- player v is also a buyer of bandwidth from its neighbors.

Utility of player v

$$U_v(X) = \sum_{u \in \Gamma(v)} x_{uv} w_u.$$

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Market Equilibrium

Market Equilibrium is an important notion for characterizing efficient allocations in exchange economies.

Market Equilibrium $(\mathbf{p}, X)$

- Market clearance: $\forall v \in V : \sum_{u \in \Gamma(v)} x_{vu} \leq 1$;
- Budget constraint: $\sum_{u \in \Gamma(v)} w_u x_{uv} p_u \leq w_v p_v$;
- Individual optimality: each player is optimally happy for its allocation at the current price.

  Formally, the solution $X = (x_{vu})$ maximizes utility $\sum_{u \in \Gamma(v)} x_{uv} \cdot w_u$ for each player v, subject to $\sum_{u \in \Gamma(v)} w_u x_{uv} p_u \leq w_v p_v$ and $x_{uv} \geq 0$.

How to find an allocation X satisfying:

- the Proportional response protocol;
- market equilibrium's allocation.

Bottleneck Decomposition

How to find an allocation X satisfying:

- the Proportional response protocol;
- market equilibrium's allocation.

Bottleneck Decomposition

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Notations:

- $w(S) = \sum_{v \in S} w_v$ for any $S \subseteq V$;
- $\Gamma(S) = \cup_{v \in S} \Gamma(v)$.
- $\alpha(S) = \frac{w(\Gamma(S))}{w(S)}$: α-ratio of S.
- B is a bottleneck of G, if $\alpha(B) = \min_{S \subseteq V} \alpha(S)$.

NOTE: $\alpha(B) \leq 1$ for bottleneck B.

Bottleneck Decomposition

Maximal Bottleneck

B is the maximal bottleneck of G, if and only if

- B is a bottleneck of G;
- $\forall\ \overline{B}$, and $B \subset \overline{B} \subseteq V$, $\alpha(\overline{B}) > \alpha(B)$.

$(B, \Gamma(B))$ is called a maximal bottleneck pair, which is actually unique. (EXERCISE)

Bottleneck Decomposition

Input: Undirected graph $G = (V, E)$;
Output: Find the bottleneck decomposition of G.

1. let $V_1 = V$, $G_1 = G$ and $i = 1$;
2. if G_i is not empty, then
3. find the maximal bottleneck pair (B_i, C_i) with $\alpha_i = \frac{w(C_i)}{w(B_i)}$ of G_i where $C_i = \Gamma(B_i) \cap V_i$;
4. $V_{i+1} = V_i - (B_i \cup C_i)$ and $G_{i+1} = G[V_{i+1}]$, $i := i + 1$ go to 2;
5. else
6. output $k = i$ and $\mathcal{B} = \{(B_1, C_1), \cdots, (B_k, C_k)\}$.

The bottleneck decomposition of G can be solved by the parametric maximum flow in a polynomial time. (EXERCISE)

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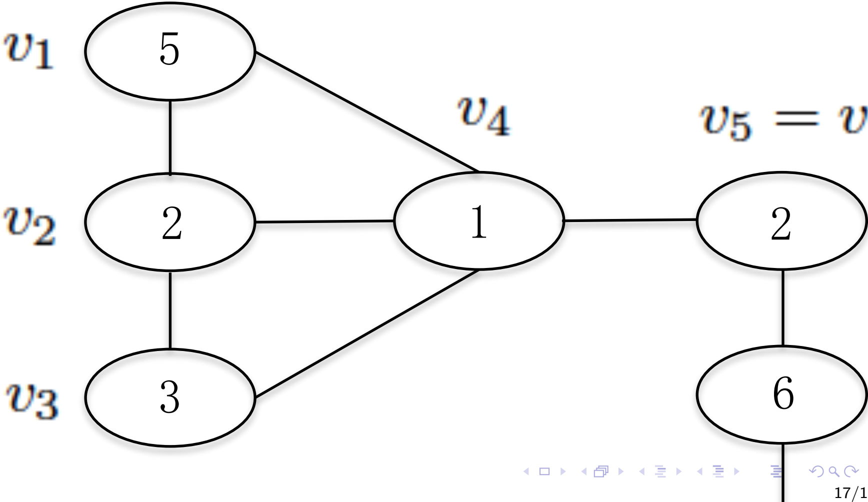

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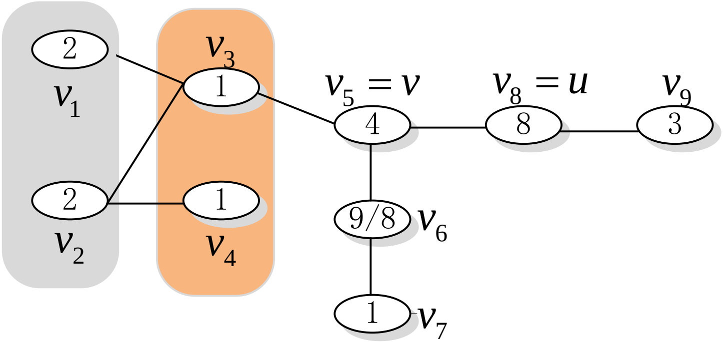
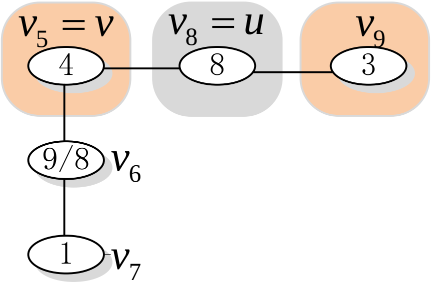


$$\alpha_1 = \frac{1}{2}$$

.

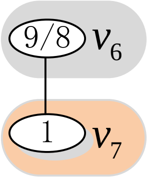
Bottleneck Decomposition



$$\alpha_2 = \frac{7}{8}$$

.

Bottleneck Decomposition



$$\alpha_3 = \frac{8}{9}$$

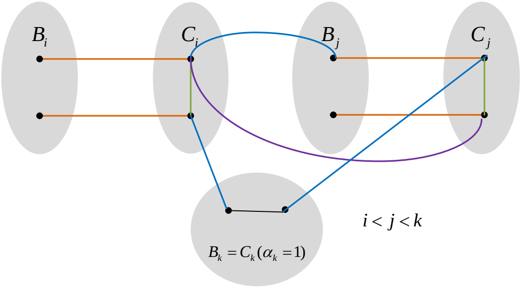
.

Bottleneck Decomposition

Property

- $0 \leq \alpha_1 < \alpha_2 < \cdots < \alpha_k \leq 1$;
- if $\alpha_i < 1$, then B_i is independent and $B_i \cap C_i = \emptyset$;
- if $\alpha_i = 1$, then $i = k$ and $B_i = C_i$.

Bottleneck Decomposition



Edge Property

- there is no edge between B_i and B_j, $i \neq j = 1, 2, \cdots, k$;
- there is no edge between B_i and C_j, where $j > i$

Bottleneck Decomposition $\Rightarrow$ a Market Equilibrium

Price vector:

$$p_u = \begin{cases} \alpha_i w_u & \text{if } u \in B_i; \\ w_u & \text{if } u \in C_i. \end{cases}$$

Allocation:

Let $\widehat{x}_{uv}$ be the amount of bandwidth u uploads to v.

- $\alpha_i < 1$: consider the bipartite graph $\widehat{G}_i = (B_i, C_i; E_i)$. By the max-flow min-cut theorem, there exist $\widehat{x}_{uv} \geq 0$ for $u \in B_i$ and $v \in C_i$, such that
  - $\sum_{v \in \Gamma(u) \cap C_i} \widehat{x}_{uv} = w_u$;
  - $\sum_{u \in \Gamma(v) \cap B_i} \widehat{x}_{uv} = w_v / \alpha_i$.

  Let $\widehat{x}_{vu} = \alpha_i \widehat{x}_{uv}$, $\Rightarrow \sum_{u \in \Gamma(v) \cap B_i} \widehat{x}_{vu} = w_v$.

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Allocation:

- $\alpha_i = 1 \Rightarrow B_k = C_k$: construct a bipartite graph $\widehat{G} = (B_k, B'_k; E'_k)$ where B'_k is a copy of B_k. If $(u, v') \in E'_k$ iff $u, v \in B_k$ and $(u, v) \in E(B_k)$.

  By Hall's theorem, for each edge $(u, v) \in E(B_k)$, there exist $\widehat{x}_{uv'} \geq 0$ such that $\sum_{v' \in \Gamma(u) \cap B'_k} \widehat{x}_{uv'} = w_v$.
- For any other edge, $(u, v) \notin (B_i \times C_i) \cup (C_i \times B_i)$, define $\widehat{x}_{uv} = 0$.

For this allocation, all available bandwidth are uploaded along edges in $B_i \times C_i$, $i = 1, 2, \cdots, k$.

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Proposition (Wu and Zhang, STOC'07)

$(\mathbf{p}, X)$ is a market equilibrium and X is a proportion response protocol. For any player v,

$$U_v = \begin{cases} w_v \cdot \alpha_i & \text{if } v \in B_i; \\ w_v / \alpha_i & \text{if } v \in C_i. \end{cases}$$

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Property of α-ratio

$0 \leq \alpha_1 < \alpha_2 < \cdots < \alpha_k \leq 1.$

$\Downarrow$

- If v is in B-class, i.e. $v \in B_i$, then $U_v \leq w_v$;
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Incentives in the Bandwidth Sharing

Allocation and Agent Incentive

Allocation mechanism:

the allocation approach of the market equilibrium from the bottleneck decomposition.

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How to Cheat?

Private Information

The neighborhood $\Gamma(v)$ is partially private: edges could be broken.

Cheating Method:

One player may remove some edges adjacent to itself.

$\Rightarrow$ The structure of P2P network is changed.

$\Rightarrow$ The bottleneck decomposition is changed.

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Remove One Edge

For simplicity

we consider the case one edge is removed.

Main Idea

Compare utility of player u (or player v) in G and $G' = G - (u, v)$ by analyzing the properties in different bottleneck decompositions.

The goal is to prove $U_u \geq U'_u$ and $U_v \geq U'_v$ since either u or v can remove this edge.

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Comparing Notations for Removing an Edge

G	G'
$\mathcal{B} = \{(B_1, C_1), \cdots, (B_k, C_k)\}$	$\mathcal{B}' = \{(B'_1, C'_1), \cdots, (B'_{k'}, C'_{k'})\}$
$\alpha_i = \frac{w(C_i)}{w(B_i)},\ i = 1, \cdots, k$	$\alpha'_i = \frac{w(C'_i)}{w(B'_i)},\ i = 1, \cdots, k'$
$V_1 = V$ and $V_{i+1} = V_i - (B_i \cup C_i)$	$V'_1 = V$ and $V'_{i+1} = V'_i - (B'_i \cup C'_i)$
$G_i = G[V_i]$	$G'_i = G[V'_i]$
$\Gamma(S) = \cup_{v \in S} \Gamma(v)$	$\Gamma'(S) = \cup_{v \in S} \Gamma'(v)$
B-class (or C-class) vertex	B'-class (or C'-class) vertex

Removed Edge Location in Decomposition

G' is obtained by removing edge (u, v).

Possible Location

- $(u, v) \in B_i \times C_j$ where $j \leq i$;
- $(u, v) \in C_i \times C_j$ where $i \leq j$.

Notations

- Assume v is in C-class if u and v are in different classes.
- Let u be in (B_{i_u}, C_{i_u}) at step $l = i_u$.
- Let v be in (B_{i_v}, C_{i_v}) at step $l = i_v$.
- $i_v \leq i_u$, if u, v are both in C-class.
- Let u (or v) appear in (B'_l, C'_l) at step $l = j_u$ (or $l = j_v$).

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Forward non-Intersection Property

Basic Lemma

For the bottleneck decomposition $\mathcal{B}$ and $\mathcal{B}'$,

- for any $1 \le t < j_*$, $B'_t \cap (\cup_{i=1}^{k} C_i) = \emptyset$;
- for ant $1 \le t < j_*$, $B_t \cap (\cup_{i=1}^{k} C'_i) = \emptyset$.

NOTE: $j_* = \min\{j_u, j_v\}$.

Above two results show that B'_t and B_t do not contain any vertex from C-class and C'-class, respectively, (only) for the case $1 \le t < j_*$.

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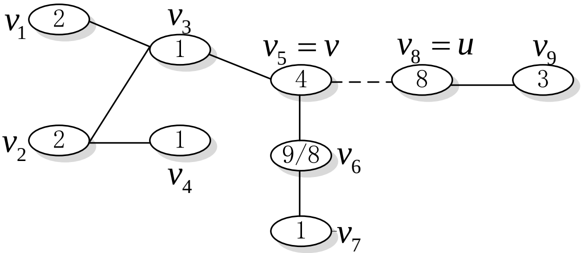

$B'_1 = \{v_1, v, v_7\}$, where v and v_7 are from C-class.

How about the general cases for B_t, $1 \leq t \leq k$ and B'_t, $1 \leq t \leq k'$?

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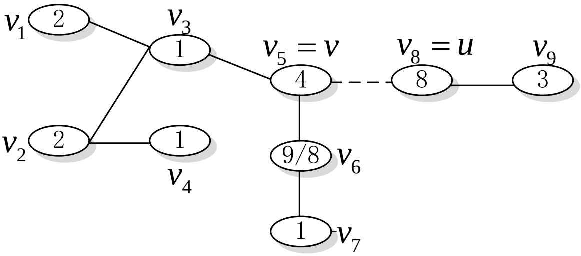

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Key Conditions for $B' \cap C = \emptyset$

Key Lemma

Consider the bottleneck decompositions $\mathcal{B}$ and $\mathcal{B}'$. If

1. $(u, v) \in B_k \times C_k$ with $\alpha_k = 1$, u and v are both in C'-class;
2. or $(u, v) \in B_i \times C_i$ with $\alpha_i < 1$, and v is in C'-class;
3. or $(u, v) \notin B_i \times C_i$, $i = 1, 2, \cdots, k$;

then for any $1 \leq t \leq k'$ with $\alpha'_t < 1$,

$$B'_t \cap \left(\cup_{i=1}^{k} C_i\right) = \emptyset.$$

Note

Key Lemma illustrates that if the conditions in Key Lemma are satisfied, then there is no C-class vertex in B'-class with $\alpha' < 1$.

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$B \cap C' = \emptyset$ Conditions

Main Lemma

Consider the bottleneck decompositions $\mathcal{B}$ and $\mathcal{B}'$. If the case that $(u, v) \in B_i \times C_i$ and u, v are both in B'-class dose not happen, then for any $1 \leq i \leq k$,

$$B_i \cap \left(\cup_{\alpha'_t < 1} C'_t\right) = \emptyset.$$

$$\Downarrow$$

Corollary

If $u \in B_{i_u}$, then u cannot be in C'-class with $\alpha' < 1$.

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Corollary

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Restriction to $B_i \cup C_i$

Theorem

If $(u, v) \notin B_i \times C_i$, $i = 1, 2, \cdots, k$, then $\mathcal{B} = \mathcal{B}'$.

One out from 16 Case Analysis

Propose the strategy-proof properties of u and v by analyzing different cases.

- u may be in B_{i_u}, C_{i_u}, B'_{j_u} or C'_{j_u};
- v may be in B_{i_v}, C_{i_v}, B'_{j_v} or C'_{j_v};

There are totally 16 cases.

Cases

$$u \in C_{i_u} \text{ and } v \in B_{i_v}$$

do not happen.

One out from 16 Case Analysis

Propose the strategy-proof properties of u and v by analyzing different cases.

- u may be in B_{i_u}, C_{i_u}, B'_{j_u} or C'_{j_u};
- v may be in B_{i_v}, C_{i_v}, B'_{j_v} or C'_{j_v};

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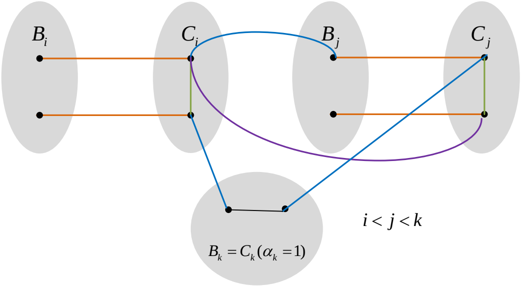
Incentive Compatibility in the Bandwidth Sharing



- $u \in C_{i_u}, v \in C_{i_v}$ and $u \in B_{i_u}, v \in C_{i_v}$ with $i_u > i_v$, then $(u, v) \notin B_i \times C_i$.

$$\Downarrow$$

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$$U_u = U'_u \quad U_v = U'_v.$$

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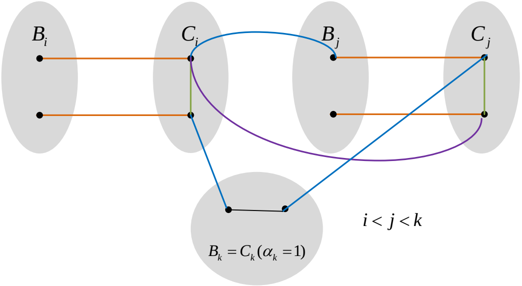

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Incentive Compatibility in the Bandwidth Sharing

- $u \in B_{i_u}, v \in B_{i_v}, \Rightarrow u \in B_k,\ v \in B_k$ and $\alpha_k = 1$.
- $u \in B_k, v \in B_k$ with $\alpha_k = 1 \Rightarrow U_u = w_u$ and $U_v = w_v$;
- $u \in B'_{j_u} \Rightarrow U'_u \leq w_u$ and $v \in B'_{j_v} \Rightarrow U'_v \leq w_v$

For case $u \in B_k, v \in B_k$ with $\alpha_k = 1$ and $u \in B'_{j_u}, v \in B'_{j_v}$,

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Incentive Compatibility in the Bandwidth Sharing

Corollary

If $u \in B_{i_u}$, then u cannot be in C'-class with $\alpha' < 1$.

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Incentive Compatibility in the Bandwidth Sharing

Lemma

If $u \in B_i, v \in B_i, u \in B'_{j_u}, v \in B'_{j_v}$, then $U_u \geq U'_u$ and $U_v \geq U'_v$.

Lemma

If $u \in B_i, v \in B_i, u \in B'_{j_u}, v \in C'_{j_v}$, then $U_u \geq U'_u$ and $U_v \geq U'_v$.

Incentive Compatibility in the Bandwidth Sharing

Main Result

Given the bandwidth allocation mechanism obtained by the market equilibrium from the bottleneck decomposition, no agent has incentive to cheat by cutting any incident edge.

Conclusion

Discuss the issue of possible cheating strategies of agent with respect to the proportional response mechanism for the application of bandwidth sharing.

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No player could gain eventually, if it removes one or more of its edge from the network environment.

- Several interesting concepts, market equilibrium, truthful mechanism, and proportional response protocol from different domains join together in this model.

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