Lecture 3: Output Sensitive Algorithm

Xiaotie Deng

AIMS Lab
Department of Computer Science
Shanghai Jiaotong University

September 23, 2017

Convex Hull Algorithm

2D Convex Hull

- Input: $D = \{\vec{d_i} = (x_i, y_i), i = 1, 2, \cdots, n\}$.
- Output: $H = \{\vec{h^j} = (x^j, y^j), j = 1, 2, \cdots, m\}$ such that $< \vec{h^1}, \vec{h^2}, \cdots, \vec{h^m} >$ are the vertices on the convex hull of D in the clockwise order.
- Properties of 2D convex hull
  - $\forall \vec{d_i} \exists 1 \leq j, k, l \leq m; \alpha, \beta, \gamma, \vec{d_i} = \alpha \vec{h^j} + \beta \vec{h^k} + \gamma \vec{h^l}$ such that $(\alpha, \beta, \gamma) \geq 0, \alpha + \beta + \gamma = 1$.
  - No vertex $\vec{h^j}, j = 1, 2, \cdots, m$, can be represented by a convex combination of two different points in ConvexHull(D).

Find a 2D Convex Hull:Divide & Conquer(Preparata and Hong 1977)

- Algorithm
  - Find the median of x-coordinates of all points x^*
  - Find the left convex hull and the right convex hull recursively
  - Find the upper/bottom tangent of left convex hull and right convex hull.
- Complexity
  - $T(n) = C * n + 2 * T(n/2) + 2 * Tangent(n)$
  - $T(n) = O(n \log n)$ if $Tangent(n) = O(n)$.

Marriage before Conquest(Kirkpatrick and Seidel 1986)

- Algorithm
  - Find the median of x-coordinates of all points x^*.
  - Find the upper/bottom tangent t$(\vec{u_l}, \vec{u_r})$/ t$(\vec{b_l}, \vec{b_r})$ of left points and right points.
  - Find upper and bottom tangents the points left to u_l/b_l recursively
  - Find upper and bottom tangents the points right to u_r/b_r recursively
- Complexity for the upper hull with h boundary edges
  - $T(n,h) \leq T(n,h_1) + T(n,h_2) + tangent(n/2, n/2)$
  - $T(n) = O(n \log h)$ if $Tangent(n/2, n/2) = O(n)$.

Tangent Algorithm

2D Tangent Computing

Formulation as a 2-variable linear programming

- $\min_{a,b} a * x_{median} + b$
- s.t. $y_i \leq ax_i + b$

Note: Here the two variables are a and b.

Solve 2-variable linear programming

- Randomized algorithm by clarkson
  - https://www.computer.org/csdl/proceedings/focs/1988/0877/00/021
- Deterministic algorithm by megiddo
  - N. Megiddo Linear programming in linear time when the dimension is fixed J. Assoc. Comput. Mach., 31 (1984), pp. 114128

Assignments: Submit answers to 5 problems

- Complete the proof $T(n) = O(n \log h)$ and find the best constant in big-O.
- Consider a set of 2D linear constraints $\{a_i x + b_i y \leq c_i, i = 1, 2, \cdots\}$. Given a point (x^*, y^*) How do you prove it satisfies all the constraints or find a violating inequality?
- What is the time complexity of the above question?
- Consider a computer system of memory size $\sqrt{n}$ and hard disk size n. How do you maintain a database which always maintains the operations of finding-median, insertion and deleting median operations. Or do it with the best complexity you can achieve.

Assignments II

- Design a streaming algorithm to find the sorted list of n numbers following the negative exponential distribution.
- Consider a series-parallel graph, design your database for shortest path query on this graph
- Given railway schedule of trains, design your database for the best arrival time query
- Consider any interesting query of the above train problem, show your solution.
- How to do handle delays of trains in updating your database?