Game Theoretical Methodology and Technique for Internet Protocols

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1. Discrete Fixed Point Computation
2. Discrete Fixed Point in 2D
3. Two Player Nash Equilibrium

Discrete Fixed Point Computation

Continuous Fixed Point

A function $f : R^n \rightarrow R^n$ has a fixed point if there is an $x \in R^n$ such that $f(x) = x$.

- Continuous Fixed Point: $f(x) : [0,1] \rightarrow [0,1]$ continuous.
- Discrete Fixed Point: $g_1(x) : \{1, \cdots, n\} = N \rightarrow N$.
- Discrete Fixed Point: $g_2(x) : N \rightarrow \{\pm 1, 0, -1\}$.
- Discrete Fixed Point: $g_3(x) : N \rightarrow \{0, 1\}$.

Assignment

Let $f(\cdot)$ be continuous. Is there always a fixed point in the following? Prove it or give a counter example.

1. Continuous Fixed Point $f(x) : [0,1] \to [0,4]$, $f(0) = 1$, $f(1) = 3$.
2. Continuous Fixed Point $f(x) : [0,1] \to [0,1]$, $f(0) > 0$, $f(1) < 1$.
3. Continuous Fixed Point $f(x) : [0,1] \to [1/3, 1/2]$
4. Continuous Fixed Point $f(x) : [0,1) \to [0,1)$.
5. Continuous Fixed Point $f(x) : [0,\infty) \to [0,\infty)$

Discrete Fixed Point

Is there a fixed point $x^* \in N$ and $g_1(x^*) = x^*$ if the following statements are true.

1. $g_1(x) : \{1, \cdots, n\} = N \to N$.
2. Discrete Continuity: $g_1(x \pm 1) \in \{g(x), g(x) \pm 1\}$.

Discrete Fixed Point

Is there a zero point $x^* \in N$ such that $g_2(x^*) = 0$ if the following statements are true.

1. $g_2(x) : \{1, \cdots, n\} = N \to \{0, -1, 1\}$.
2. $\forall x : x + g_2(x) \in N$.
3. Discrete Continuity: $g_2(x) * g_2(x+1) \geq 0$.

Discrete Fixed Point

Is there a zero point $x^* \in N$ and $g_3(x^*) = 0$ and $g_3(x^* + 1) = 1$ if the following statements are true.

1. $g_2(x) : \{1, \cdots, n\} = N \to \{0, 1\}$.
2. $\forall x : x + g_3(x) \in N$.
3. $g_3(0) = 0$ and $g_3(n) = 1$.

Assignment

Is there an adjacent pair (x^*, x^*+1) and $g_4(x^*) * g_4(x^*+1) = -1$ if the following statements are true.

1. $g_4(x) : \{1, \cdots, n\} = N \to \{-1, 1\}$.
2. $g_4(1) + g_4(n) = 0$.

Discrete Fixed Point Computation in 2D

2D Fixed Point

- Direction Preservation Zero Point
- SPERNER's triangle
- Tucker Edge

Oracle Complexity

- Index and degree
- Binary Search
- Path following

Polymial Computable Function Model Complexity

- PPAD
- PPA
- Odd index boundary

Two Player Nash Equilibrium

Pure Nash Equilibrium

- Prisoner's Dilemma

strategies	stays silent	betrays
stays silent	-1,-1	-5, 0
betrays	0,-5	-3,-3

- Game of Chicken

$$RowPlayerPayoffs \begin{pmatrix} -2 & 1 \\ -1 & -1 \end{pmatrix} \quad ColumnPlayerPayoffs \begin{pmatrix} -2 & -1 \\ 1 & -1 \end{pmatrix}$$

- Battle of Sexes

events	opera	football
opera	4,2	0,0
football	0,0	2,4

Mixed Nash Equilibrium

- Rock-Paper-Sissors

$$\begin{array}{cc} & \begin{array}{ccc} \mathit{rock} & \mathit{paper} & \mathit{scissors} \end{array} \\ \begin{array}{c} r \\ p \\ s \end{array} & \begin{bmatrix} 0,0 & -1,1 & 1,-1 \\ 1,-1 & 0,0 & -1,1 \\ -1,1 & 1,-1 & 0,0 \end{bmatrix} \end{array}$$

- Matching Pennies Game

	head	tail
head	1,-1	-1,+1
tail	-1,+1	+1,-1

Best responses and Nash Equilibrium

- Given opponent i's strategy x_i, the best response of $2-i$ is $\mathrm{BR}_{2-i}(x_1, x_2)$, $i = 1, 2$.
- Let (x_1', x_2') =BR$(x_1, x_2) = (\mathrm{BR}_1(x_2),\mathrm{BR}_2(x_1))$.
- $\mathrm{BR}(\vec{x}) : S_2 \Rightarrow S_2$ maps $S_2 = \{x \geq 0, x_1 + x_2 = 1\}$ into itself.
- BR has a fixed point x^* by the fixed point theorem.
- Assignment: there is an error in the proof, how to fix it?

Computation

- Lemke-Howson's Algorithm
- PPAD solution model
- Nash Computation

Self Study Requirement

- Become an Expert in the Subject of One Lecture
- Criteria:
  - Basic: understand all the proofs and ready to help other students to prepare for the midterm/final exam (4%)
  - Middle level: Acquire an appropriate dataset for big data analysis (input size $<$ memory space $<$ output database size $<$ hardisk space, 3%)
  - Advanced: Understand one interesting new method and prepare a ppt file for next year students (2%)
  - Excellency: Propose/implement an appropriate algorithm or solve a problem at research frontier (1%)